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# STIFFNESS OF COMPOSITE CFST COLUMNS

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## ABSTRACT

Concrete-filled steel tubes (CFSTs) columns are used extensively in modern buildings and a number of different expressions are available to assess their flexural stiffness for use in design. The flexural stiffness is used in the determination of the elastic critical buckling load for a slender column, and it is used in analyses of frames to determine likely actions and displacements under loadings including earthquake. The different available expressions can give significantly different results.

This paper describes the background of a number of empirical relationships for composite column stiffness commonly considered for design. Also, the values of stiffness obtained are compared with those obtained from simple moment-curvature analysis. It is shown that the secant flexural stiffness of columns is almost independent of the axial loading for a specific section in the moment curvature analyses, and this is consistent with some AISC, ACI and NZS3404 design recommendations. Some AISC design provisions developed to determine the member compressive axial force result in a greater computed stiffness than the other methods. Also, for the limited range of analyses conducted with section slenderness near the NZS3404 slenderness limit, NZS3404 methods, which are similar to ACI methods, gave similar results to the moment-curvature analysis. Issues relating to design are discussed, and for situations where low stiffness is critical, the NZS3404 method and the ACI method were considered to be appropriate/conservative for calculating flexural stiffness for design.

## Introduction

Concrete-filled steel tubular (CFST) members utilise the composite interaction between steel and concrete material to develop more efficient members than those from steel or concrete alone. The concrete infill is confined by the steel tube, resulting in a tri-axial state of compression that increases the strength and strain capacity of the concrete. The peripheral steel can perform most effectively in tension, while the steel in compression is braced by the concrete infill, delaying the occurrence of local buckling of the steel tube. The result is a composite member with high axial, flexural, and shear stiffness and strength in both directions for the quantity of steel and concrete provided. Such CFST members are becoming popular in New Zealand, especially in moment-resisting frames, where columns are designed to resist lateral forces in the building in two horizontal directions, and also for slender columns. While established methods exist to estimate strength, equations available to estimate the stiffness give widely differing answers. If column stiffness is overestimated, this may result in an unsafe design with the axial strength being overestimated. Also, the frame will be more flexible than anticipated, possibly causing discomfort during serviceability conditions, and seismic lateral drifts of frame will be underestimated with implications for building stability and stair seating length.

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While methods to estimate stiffness are provided in NZS3404, these are similar to a method by ACI, and it is not clear how relevant these may be for composite members. Moreover, in one table of the Guide (Ziemian, 2012), which is used for the design of steel structures, only the AISC equations are listed. Furthermore, Denavit (2012) lists a number of methods from different sources without describing their background or use. It perhaps intuitive to believe that the stiffness of a CFT column should increase with increasing axial force, yet most methods to obtain stiffness do not consider this effect. As a result, it is not clear to many what method should be used for in structural design, or even if the methods which are currently available are reasonable.

It may be seen from the above discussion that there is a need to understand why different estimates of CFST stiffness are available, and to know what should be used in design.

This paper attempts to address this need by seeking answers to the following questions:

1) What studies and recommendations are available to determine CFST stiffness and what is their background?
2) How do they compare with the results from moment-curvature analysis for a typical column?
3) How does computed axial strength change in a typical axially loaded column as a result of a different computed stiffness?
4) How does stiffness change with a different section slenderness ($d_0/t$) ratio?
5) What stiffness expressions should be used for design?

## Previous Studies to Determine (CFST) Flexural Stiffness and Resulting Recommendations

Schiller et al (1994) showed that the initial flexural stiffness of a CFT column is simply the uncracked concrete and steel contributions. The stiffness decreases as cracking occurs.

AISC (2010) specifies Eq. 1 and 2 to calculate flexural stiffness of composite steel filled tube sections where $E_s$, $I_s$ , and $A_s$ are the elastic modulus, second moment of area, and area of the steel and $E_c$, $I_c$, and $A_c$ are the elastic modulus, second moment of area, and area of the concrete at the section. This method is henceforth referred to as the AISC method. The parameter $C_3$ is less than unity as shown in Eq. 2 which means only part of the concrete is active. These equations are intended to calculate nominal strength of a column without bending moment ($P_n$) as an anchor point on an interaction equation. Normally $P_n$ would include incorporation of an effective length (Hajjar, 2015; Leon, Kim and Hajjar, 2007; and Leon and Hajjar, 2008).

$$EI_{\mathrm{eff}} = E_{\mathrm{s}}I_{\mathrm{s}} + C_{\mathbf{3}}E_{\mathrm{c}}I_{\mathrm{c}} \tag{1}$$

$$(AISC\ 360-10\ Eq.I2-12)$$

$$C_{\mathbf{3}} = 0.6 + 2\left(\frac{A_{\mathrm{s}}}{A_{\mathrm{c}}+A_{\mathrm{s}}}\right) \leq 0.9 \tag{2}$$

$$(AISC\ 360-10\ Eq.I2-13)$$

ACI (2008) provisions  are gives in Eq. 3 for flexural stiffness of composite steel filled tube sections, where $\beta_d$ is the sustained load factor taken as the ratio of the maximum factored axial dead load to the total factored axial load (for the type of column studied) and is always positive. This method is henceforth referred to as the ACI method. For short term loads $\beta_{\boldsymbol{d}}$=0.0. ACI is primarily concerned with concrete (including reinforced concrete) structures. Stability is generally not treated as a major concern for such structures and the stiffness is generally used to determine the displacement response during a frame analysis. For this purpose, a lower EI would be expected than for the AISC axial loading case above as more cracking is expected as the structure can reach significant displacements before yield.

$$EI_{\mathrm{eff}} = E_{\mathrm{s}}I_{\mathrm{s}} + \left(\frac{0.2E_{\mathrm{c}}I_{\mathrm{g}}}{1+\beta_{\mathrm{d}}}\right) \tag{3}$$

$$(ACI\ 318-08\ Eq.10-23)$$

Roeder et al. (2010) looked at 50 tests of circular CFSTs subjected to combined axial and flexural loading by a range of researchers. In order to provide an evaluation of the effective stiffness in this study, graphical interpretation of the force-deflection curves were used to estimate the initial effective stiffness of the experimental specimens. Eq.4 provides the method used to calculate the effective stiffness from the force-deflection response of the experimental specimens. Fig. 1 illustrates the process used to graphically interpret the initial stiffness from a force-deflection curve. The initial slope of the curve was linearly extrapolated to a resistance value, and a straight line was drawn down to the X-axis in order to determine the deflection at that point. These values were then used in Eq. 4 to determine the effective initial stiffness of the CFST specimen. Eq. 5 was recommend as an improved initial stiffness expression where *C*' is an empirical factor. The value of C', obtained from the test data shown in Fig. 2 (Bishop 2009) indicates a large variation of flexural stiffness with axial compressive load, and no clear trends with axial force (Roeder 2010). Nevertheless, they fitted a line to the data and obtained the average expression in Eq. 6 and 7 with a linear regression $R^2$ value of 0.303. This equation has also been included in Table 2.4 of the thesis of Denevit (2012) and there is no indication of the variation from that in the equations. This method is henceforth referred to as the Roeder method where *L*, *P*, *Δ*, *and* $f'_c$, are the length of CFST Column, applied point load , measured specimen deflection, and compressive strength of confined concrete, respectively and $P/P_0$ is the axial load ratio which is not permitted to be greater than 0.75

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$$EI_{\text{eff-experimental}} = \frac{P.L^3}{3\Delta_{\max}} \tag{4}$$

$(Bishop, 2009\ Eq.5-17)$

$$EI_{\text{eff}} = E_s I_s + C' E_c I_c \tag{5}$$

$(Bishop, 2009\ Eq.5-21)$

$$C' = 0.15 + \frac{P}{P_0} + 2\frac{A_s}{A_c + A_s} < 0.9 \tag{6}$$

$(Bishop, 2009\ Eq.5-20)$

$$P_0 = F_y A_s + 0.95 f'_c A_c \tag{7}$$

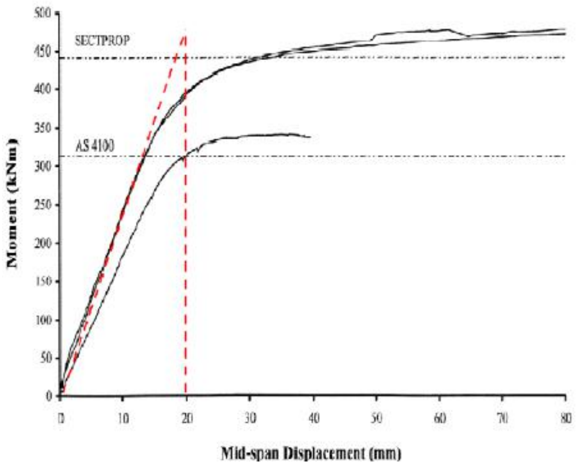


Fig.1. Graphical Interpretation of the experimental initial stiffness for a CFST element (Bishop, 2009, Fig 5-64)

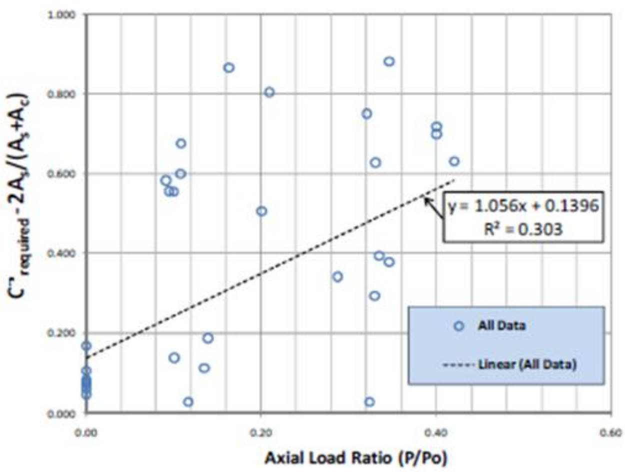


Fig 2. Required concrete stiffness factor to fit experimental data (Bishop, 2009, page 216, Figure 5-69)

Denevit and Hajjar (2014) also indicated that there should be an increase in stiffness with axial force and reduction with increasing moment. Eqs. 8-10 were used to represent the secant stiffness required for elastic frame analysis for design. The basis for these equations seems promising, but further discussion of them is not conducted in this paper as they are relatively new and are not commonly considered for design in NZ.

$$EI_{\text{elastic}} = E_{\text{s}}I_{\text{s}} + C_5 E_{\text{c}}I_c \qquad \text{CFST} \qquad (8)$$

$$(Denavit, 2012\ Eq. 6-10)$$

$$C_5 = 0.99 - 0.73\frac{M}{M_n}\left[1 - 3.47\frac{P}{P_{no}}\right] \leq 1.00 \qquad \text{CCFST} \qquad (9)$$

$$(Denavit, 2012\ Eq. 6-11a)$$

$$C_5 = 1.01 - 0.90\frac{M}{M_n}\left[1 - 3.39\frac{P}{P_{no}}\right] \leq 1.00 \qquad \text{RCFST} \qquad (10)$$

$$(Denavit, 2012\ Eq. 6-11b)$$

Tikka and Mirza (2006) also developed a nonlinear equation for *EI* for use in design of slender composite columns as an alternative to the existing ACI equations for $EI_{\text{eff}}$. Eq. 11 is the proposed design expression for short-term EI of slender composite steel-concrete columns under major axis bending where $I_{\text{g}}$, $I_{\text{rs}}$, and $I_{\text{ss}}$ are the moment of inertia of gross concrete section , of longitudinal reinforcing bar, and of structural steel section taken about centroidal axis of composite cross section; and *e*, *h*, and *l* are defined as end eccentricity, overall thickness of cross section perpendicular to the axis of bending, and unsupported length of the column, respectively.

$$EI = \left[0.47 - 3.5\frac{e}{h}\left(\frac{1}{1 + 9.5\frac{e}{h}}\right) + 0.003\frac{l}{h}\right]E_c\left(I_{\text{g}} - I_{ss}\right) + 0.8E_{\text{s}}(I_{\text{ss}} + I_{\text{rs}}) \qquad (11)$$

Comparison of the proposed design equation (Eq. 11) with ACI design equation (Eq.3) showed that the average stiffness ratios for the ACI design equations tend to be more conservative than that for the proposed design equation. However, the one-percentile stiffness ratios obtained for ACI design equations and for the proposed design equation are approximately the same.

NZS 3404 (1997) provides calculation of composite section capacity for a concrete filled steel tubular column in accordance with NZS 3101 or an “appropriate limit state design procedure” (Clause 13.8.3.3). The commentary recommended a UK based approach. The proposed equation for flexural rigidity is given by Eq. 12, which is the same as the ACI method.

$$EI = \frac{0.2E_{\text{c}}I_{\text{g}}}{1 + \beta_{\text{d}}} + E_{\text{s}}I_{\text{s}} \qquad (12)$$

$$(NZS\ 3101, 2006\ Eq. 10-25)$$

### Comparison of Some Existing Expressions with Moment-Curvature Analysis

Moment-curvature analysis was conducted using a cross-sectional fibre model to evaluate the flexural stiffness of circular CFST columns subjected to a constant axial load and monotonically increasing moment to given another expression for stiffness. The section slenderness parameter is calculated in accordance with NZS 3404 (1997), as shown in Eq.13 where $d_0$is the outside diameter of the section is; t is the steel tube thickness, and $f_{\text{y}}$ is the steel yield stress. For $f_{\text{y}}$ = 300 MPa, $d_0$ = 610mm and t = 6.4mm the section slenderness parameter is 114.4. NZS 3404 (1997) clause 13.8.3.2b states that concrete-filled circular sections must have a section slenderness, $\lambda_{\text{e}}$, not exceeding 120. It does not explicitly state whether compression or bending is considered here. The clause also refers to Table 5.2 where a maximum slenderness of 120 is specified for hollow circular sections subject to bending. This value is significantly more than the acceptable section slenderness limit for hollow circular members in compression of 82 as specified in Table 6.2.4.

$$\lambda_s = \left(\frac{d_0}{t}\right)\left(\frac{f_y}{250}\right) \quad (13)$$

Stress-strain curves for the steel tube and concrete infill of the CFST column in compression follow Fujimoto et al. (2004) as shown in Figs. 3 and 4.

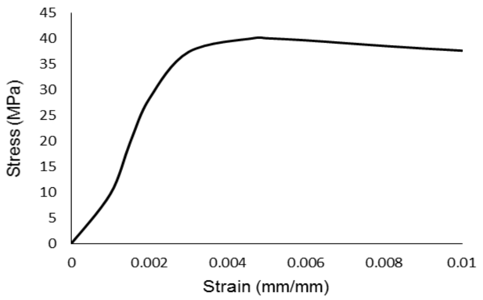


Fig 3. Effective stress-strain curve for confined concrete of circular CFSTs

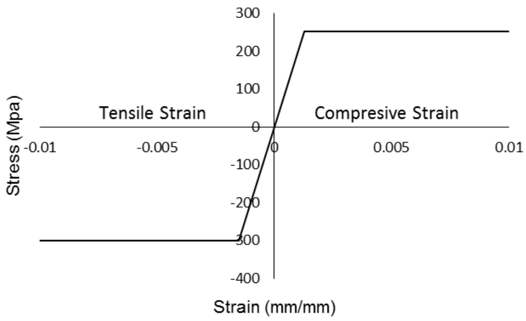


Fig 4. Effective stress-strain curve for steel tube of circular CFSTs

The moment-curvature analysis was conducted using the program SAP 2000 version 17. The major assumptions used in determining the moment curvature analysis were: (1) the confining effects of the circular steel tubes were considered based on the research results presented by Sakino et al. (1998) for axially compressed confined concrete; (2) the stress-strain relationship of a circular tube was assumed to be elastic-perfectly plastic; (3) stiffness was evaluated ignoring concrete in tension. Assumptions (2) and (3) are conservative. The moment-curvature analysis was initially verified with selected cross-sections from Priestley et al. (2007).

Fig. 5 provides moment-curvature curves resulting from analysis for four levels of axial load ratios. Only initial part of the moment-curvature curves has been included, to enable region up to, and immediately after yield point to be clearly differentiated.

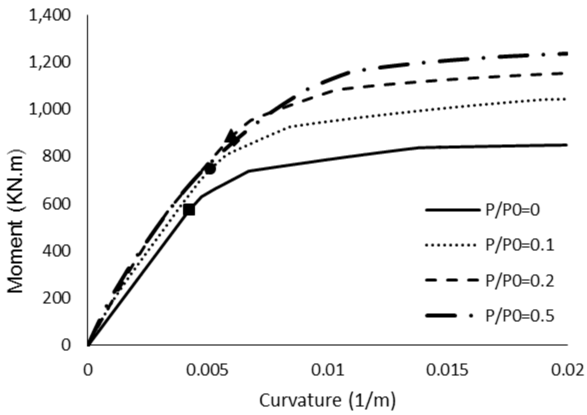


Fig 5.Moment curvature curves for CFSTs for different axial load ratios

It may be seen that the initial tangent stiffness from the origin increases slightly with axial force. However, for large moments, the secant (effective) stiffness passing from the origin through the first yield point remains near constant. First yield is defined as the point on the moment-curvature response curve when the extreme tension steel fibre (the furthest fibre from the neutral axis) attains yield strain, or when the extreme concrete compression fibre (again at maximum distance from the neutral axis) attains a strain of 0.003, whichever occurs first. It may be seen that the secant stiffness is almost constant for axial force.

The stiffness was normalized by the initial uncracked section stiffness and compared to the cross-sectional stiffness computed by the AISC, ACI and Roeder methods as shown in Fig.6. The moment-curvature analysis conducted is designated as “SAP”. Note that $\beta_d$ is considered as zero (short term loading) in order to make a comparison between the results.

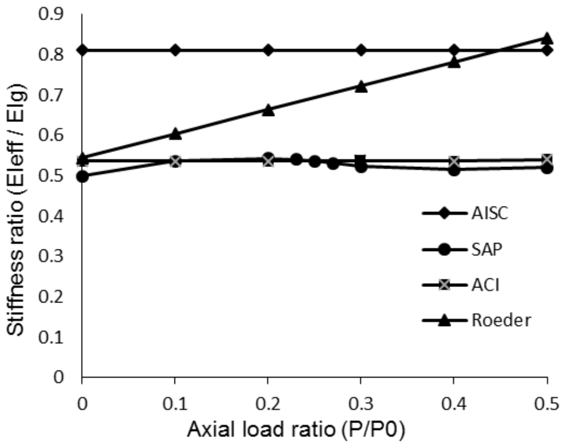


Fig 6. Comparison of Effective Stiffness Ratio with AISC, ACI and Roeder equations ($\lambda_s$=114.4)

The AISC approach gives greater values than for the other methods as shown in Fig. 6. The reason for this is that the AISC approach is intended to use the axial force capacity with an effective length factor as described above. The other methods are for elastic member analysis and are based on secant stiffness.

The Roeder method provides a linear transition between ACI and AISC values based on the axial load ratio. Again, this curve is based on Fig. 2, and there is a large amount of scatter, so it is not clear how much weighting should be given to this method.

The ACI method is constant with axial load. This is consistent with performed moment-curvature analysis for sections with this $d_0/t$ ratio.

It should be noted that the moment-curvature analysis provides the stiffness at a section. For a member under lateral force the maximum moment is at the column ends. Over the rest of a column, where there is a lower moment, there will be less section cracking and therefore greater effective stiffness, $EI_{eff}$. The moment-curvature analysis therefore gives a lower bound on the actual lateral stiffness of the total column at significant moment (the yield moment here) in terms of *EI*.

**Effect of EI variation on Column Strength**

To evaluate the effect of different $EI_{eff}$ values of circular CFST columns on axial strength, the following comparison was performed between the ACI and AISC equations. This was done because these were approximate bounds on *EI* obtained for the column analyzed. The NZS 3404 (1997) buckling equation was used for a section with residual stresses corresponding to $\alpha_b$ = -1, since this was the category of the steel tube listed for hot rolled tubes. The axial force ratio, $N_c/N_s$, is plotted against the nominal slenderness ratio $\lambda_n$ as shown in Fig. 7. The nominal member capacity, $N_c$. of a member with constant cross section and the nominal slenderness ratio were determined using Eqs.14, 15, and 16 following the NZS 3404 (1997) where $\alpha_c$, $k_f$ , $A_n$, $L_e$, and $f_y$, are the member slenderness reduction factor, form factor, net area of the cross section, effective length and steel yield stress respectively. The effective length ($L_e$) was calculated considering the column as a sway member fully fixed at the base and free at the top giving an effective length value of 2, and $k_f$ was calculated as 1 based on the selected section properties.

$$N_c = \alpha_c N_s \qquad (14)$$

$$N_s = K_f A_n f_y \qquad (15)$$

$$\lambda_n = \frac{L_e}{r}\sqrt{K_f}\sqrt{\frac{f_y}{250}} \qquad (16)$$

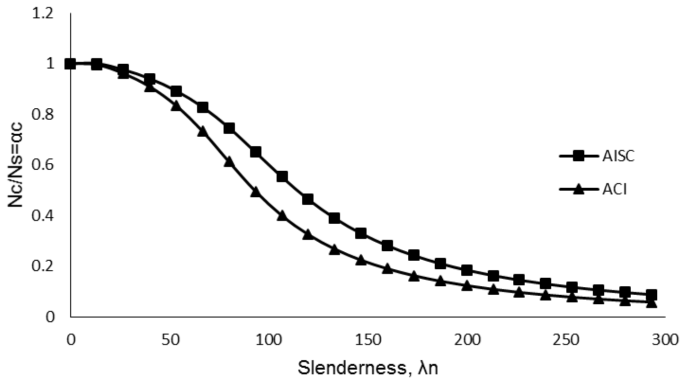


Fig 7. Comparison of NZS provision stability curve with fibre analysis curve for AISC and ACI methods of computing *EI*

As can be seen in Fig. 7, the AISC method indicates a larger buckling strength than the ACI method for the same slenderness ratio. For example, for a slender column with a slenderness of 200, the buckling strength ratios were 0.18 and 0.12 for the *EI* form the AISC and ACI methods respectively indicating that the AISC method may overestimate the ACI strength by 150% for a relatively slender column.

### Effect of Section Slenderness Ratio ($d_o/t$)

The analysis above was based on a section with a slenderness ratio of 114.4 which did not exceed the maximum local slenderness limit for yield ($\lambda_e$ = 120) given in Table 5.2 of NZS 3404 (1997). To evaluate the effect of width to thickness ratio, analysis was carried out using a section with $d_0$ (610mm), t (4.8mm) and $d_0$/t= 127, so the section slenderness parameter is 152.5. Such a section is not allowed to be used in NZS3404 but is allowed to be used in AISC (2010), Table I.1.1A. Fig. 8 compares the cross-section effective stiffness at different levels of axial loading calculated by moment curvature analysis (SAP) with AISC, ACI, and Roeder equations.
As shown in Fig. 8, the *EI* values obtained from moment-curvature analysis is about 10% less than that obtained from the ACI method, and significantly less than that of the AISC and Roeder equations.

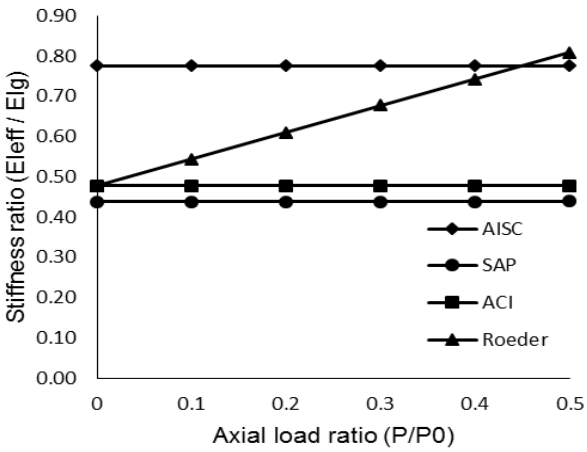


Fig 8. Comparison of analysis effective stiffness ratio with AISC, ACI and Roeder equations ($\lambda_s$=152.5)

### Considerations for Design

Some general considerations are that:

1) The flexural stiffness of a composite column required for design depends on the use of the flexural stiffness. For example, the AISC method, seeks to determine the axial load capacity of a column member with effective length considerations. Using this method is not appropriate to obtain a flexural stiffness for general frame analysis. The values of *EI* can be further affected if they are to be used

with a particular analysis method, such as the Direct Analysis method in the AISC standard. The initial flexural stiffness of an uncracked member is different from that of a cracked member, and it may be different again for a member which has been subject to significant creep, or to a number of load repetitions. The moment profile will also affect the stiffness, as will the concrete age.

2) The Roeder study shows that column stiffness from experimental studies can result in a large amount of scatter. As a result of the scatter, and lack of trends, not method should be regarded as being correct. Nevertheless, methods are required for design.
3) While the ACI method has some consideration for creep, there is no explicit consideration for shrinkage. Such shrinkage can occur and it can be exacerbated if holes are placed in the tube to allow steam to escape from a column due to fire. Shrinkage may mean that the concrete can move away from the wall of the structure and there is very little composite action, especially if there is low member curvature. Experts have different opinions as to the amount and effect of shrinkage.
4) Using a lower estimate of flexural stiffness for design is generally conservative in the following cases:
    a. For considering earthquake induced structural displacement, a lower stiffness will generally result in greater drifts, which affects structural demands and the required seating length of elements between buildings (e.g. elevated walkways) or within buildings (e.g. stairs). Therefore overestimating stiffness is non-conservative.
    b. The axial strength of a column is increased with increasing stiffness according to the Euler equation and column buckling curves. Therefore overestimating stiffness is non-conservative.

    However, in the earthquake situation, it is possible that underestimating the stiffness may in some cases result in greater inelastic demands in a structure. Nevertheless, it is the opinion of the authors that this effect is not likely to be very significant in a ductile structure so can be ignored.
5) The lowest stiffness that can be reasonably considered is to use the bare steel alone to compute the stiffness. The full steel section properties can be used if the section elastic slenderness limit is not exceeded.
6) The secant flexural stiffness obtained from moment-curvature analysis at first yield may be lower than that in realistic members for lower member curvatures. This may occur due to smaller lateral demands, and also because the cracking in the concrete is not likely to be the maximum value over the column height. It may also be non-conservative if creep and shrinkage is greater than that anticipated, or if there is slip at the concrete-steel interface reducing the composite action.
7) It is considered that secant flexural stiffness obtained from moment-curvature analysis at first yield, albeit simple and crude, may give reasonable flexural stiffness for design of realistic members where a low estimate of flexural stiffness is conservative.
8) ACI/NZS3404 equations and moment-curvature analysis gave similar estimates of stiffness, with little effect of axial force, when the section slenderness was less than 120 in the analyses shown.
9) Further work needs to be undertaken to understand likely column stiffness but a method to design is necessary in the meantime.

Based on the considerations above it is recommended that the ACI/ NZS 3404 method is appropriate when $\lambda_s \leq 120$.

## Conclusions

This study compared a number of methods to estimate circular CFST column flexural stiffness. These were methods published by ACI, AISC, NZS3404, Roeder, moment-curvature analysis, and some others. It was shown that:

1) The different methods to give significantly different column flexural stiffness values. This is because the methods have been obtained using different procedures and for different purposes. For example, the AISC method is intended to obtain the axial force capacity with an effective length factor, while some other methods, such as the ACI, NZS3404 and Roeder method are evaluating a stiffness for elastic analysis. Some methods indicate that the axial stiffness is dependent on axial force while others do not. Roeder indicated that there is considerable variation in the stiffness of actual columns.
2) Moment curvature analysis was conducted on a column member with $d_o/t$ of 95. It was found that the initial tangent $EI$ increased with axial force, but the effective $EI$, computed at the first yield of the specimen, did not significantly depend on axial force. The moment-curvature results were reasonably consistent with the ACI method and the NZS3404 method. The AISC and Roeder methods generally gave stiffnesses up to 60% greater than that of the moment-curvature method.
3) The difference between the axial strengths obtained for a member using the different methods varied as much as 150%. This has consequences for structural design. It also indicates that designers

should be aware of the background aims and assumptions of the different methods to avoid selecting values that may result in an unsafe design.

4) For a column with a larger $d_o/t$ of 127, which was greater that the allowable section slenderness limit in NZS3404, the ACI/NZS3404 stiffness was about 10% more than that from moment curvature analysis.
5) Based on the consideration that low flexural stiffness is likely to be conservative for both estimating non-seismic and seismic frame displacements and for obtaining member axial force capacities, it was recommended that the NZS3404 method be used to compute CCFT column flexural stiffness until better information is available.

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